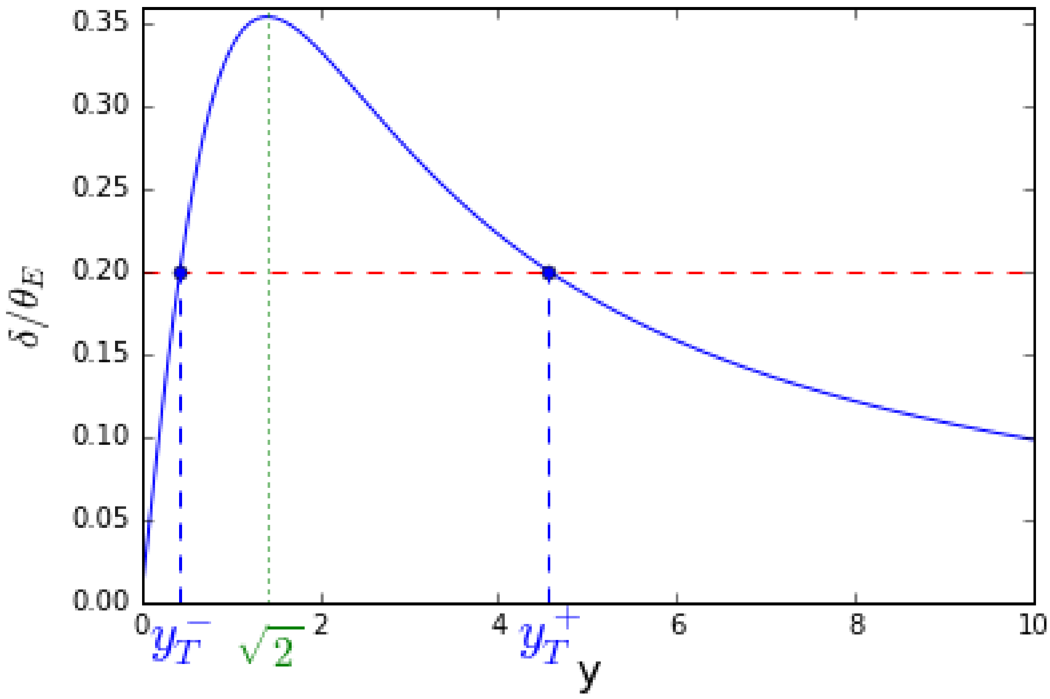
Lecture 10

June 4, 2016

1 Cross section for astrometric microlensing

In order to produce a centroid shift $\delta > \delta_T$, the source has to stay within a certain distance from the lens. This can be seen by looking at the shape of the function $\delta(y)$:

```
In [24]: import numpy as np
         import matplotlib.pyplot as plt
         %matplotlib inline

         def delta(y):
             return (y/(y**2+2))

         y=np.linspace(0,10,1000)
         d=delta(y)

         plt.plot(y,d)

         x1=[np.sqrt(2),np.sqrt(2)]
         x2=[0,10]
         plt.plot(x1,x2,":")
         plt.ylim([0,0.36])

         x1=[0,10]
         dt=[0.2,0.2]
         plt.plot(x1,dt,"--")

         plt.xlabel(’y’,fontsize=16)
         plt.ylabel(r’$\delta/\theta_E$’,fontsize=16)

         def yT1(K):
             return(K/2.0+np.sqrt(K**2/4.0-2.0))

         def yT2(K):
             return(K/2.0-np.sqrt(K**2/4.0-2.0))

         yt=[yT1(1.0/dt[0]),yT2(1.0/dt[0])]

         plt.plot(yt,dt,’o’,markersize=5,color=’blue’)
         plt.plot([yt[0],yt[0]],[0.0,dt[0]],’--’,color=’blue’)
         plt.plot([yt[1],yt[1]],[0.0,dt[0]],’--’,color=’blue’)
         plt.text(yt[1],-0.03,’$y_T^-$’,fontsize=20,horizontalalignment=’center’,color=’blue’)
         plt.text(yt[0],-0.03,’$y_T^+$’,fontsize=20,horizontalalignment=’center’,color=’blue’)
         plt.text(np.sqrt(2.0),-0.04,’$\sqrt{2}$’,fontsize=16,horizontalalignment=’center’,color=’green
Out[24]: <matplotlib.text.Text at 0x110eff590>
```



As we have seen, the function has a maximum in $y = \sqrt{2}$. Setting the threshold at $\delta/\theta_E = 1/K$, then $\delta > \delta_T$ for

$$y_T^- < y < y_T^+$$

where

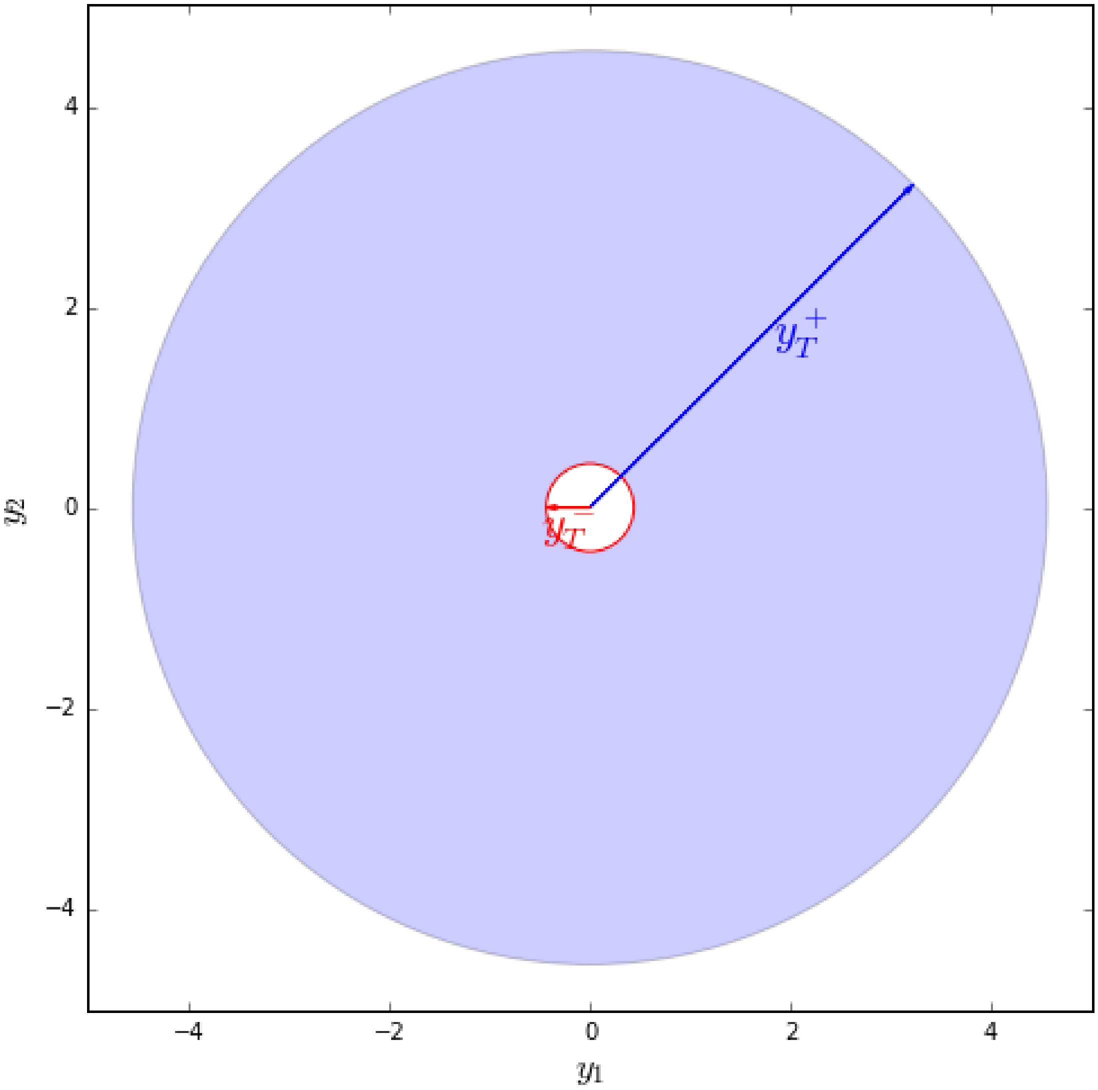
$$y_T^{+/-} = \frac{K}{2} \pm \sqrt{\frac{K^2}{4} - 2}$$

The cross section for astrometric microlensing is

$$\sigma = \pi(y_T^{+^2} - y_T^{-^2})\theta_E^2$$

```
In [63]: fig,ax=plt.subplots(1,1,figsize=(8,8))
         circle_out=plt.Circle((0,0),yt[0],color=’black’,fc=’blue’,fill=True,alpha=0.2)
         circle_in=plt.Circle((0,0),yt[1],color=’red’,fill=True,fc=’white’)
         ax.add_artist(circle_out)
         ax.add_artist(circle_in)
         ax.set_xlim([-yt[0]*1.1,yt[0]*1.1])
         ax.set_ylim([-yt[0]*1.1,yt[0]*1.1])
         plt.xlabel(r’$y_1$’,fontsize=16)
         plt.ylabel(r’$y_2$’,fontsize=16)
         ax.arrow(0, 0, (yt[0]-0.1)*np.cos(np.pi/4), (yt[0]-0.1)*np.sin(np.pi/4), head_width=0.05, head_
         ax.arrow(0, 0, -(yt[1]-0.1), 0, head_width=0.05, head_length=0.1, fc=’red’, ec=’red’)
         ax.text(yt[0]/2.0*np.cos(np.pi/4)+0.2, yt[0]/2.0*np.sin(np.pi/4),r’$y_T^+$’,fontsize=20,horizon
         ax.text(-yt[1]/2.0, -0.3,r’$y_T^-$’,fontsize=20,horizontalalignment=’center’,color=’red’)
```

```
Out[63]: <matplotlib.text.Text at 0x1184a2b10>
```



In the limit $K >> 1$, $y_T^- \to 0$, and $y_T^- \to K$. In this case, the central "hole" in the cross section can be neglected, and the cross section can be approximated as

$$\sigma \approx \frac{\pi\theta_E^4}{\delta_T^2}$$

2 Time variation of the centroid shift

The centroid shift varies with time as

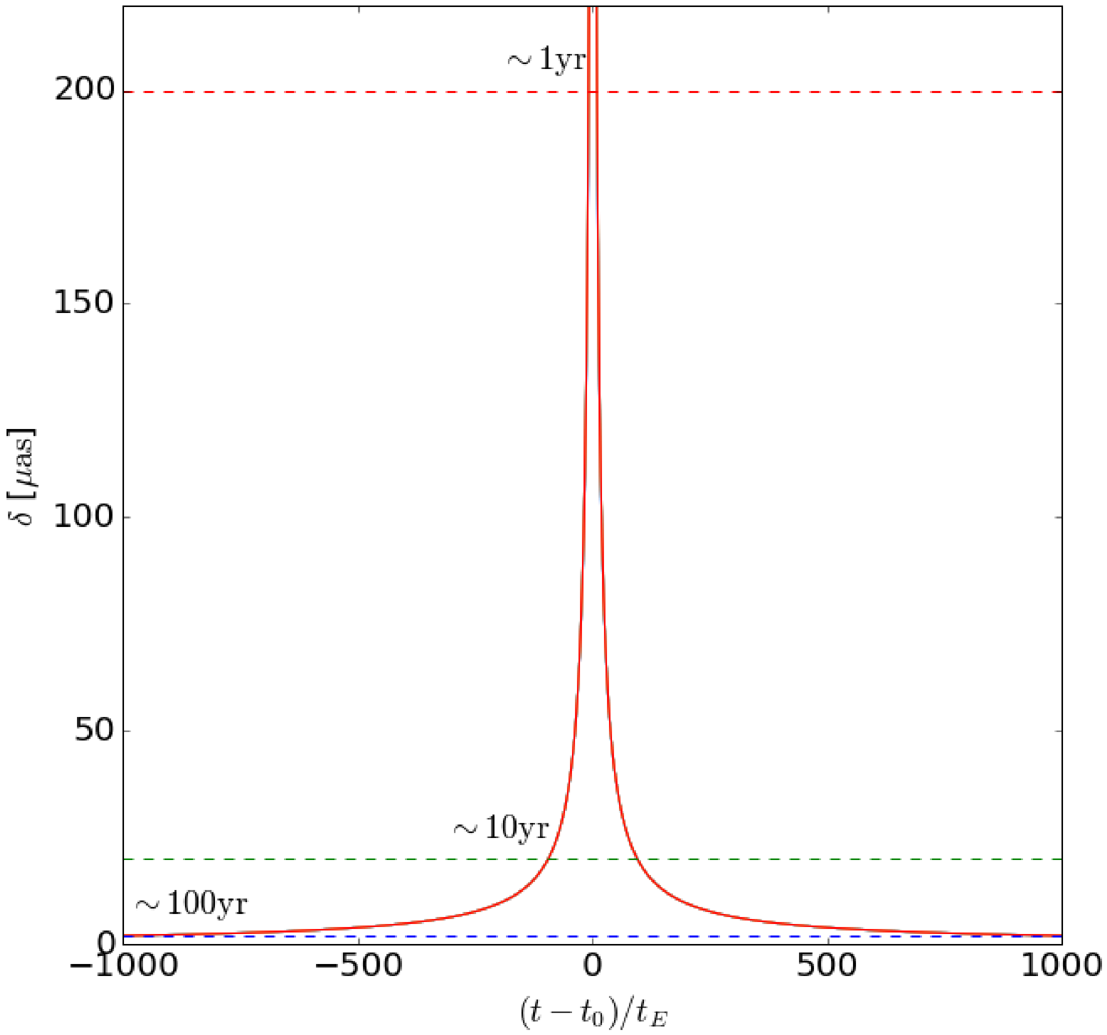
$$\delta(t) = \frac{y(t)}{y^2(t)+2}$$

The rate at which it decreases at large y is

$$\frac{d\delta}{dt} \approx -58\frac{1}{y^2}\left(\frac{v}{100\text{km/s}}\right)\left(\frac{D_L}{1\text{ kpc}}\right)\ \mu\text{as day}^{-1}$$

```
In [101]: %matplotlib inline
          from matplotlib.pyplot import cm
          import matplotlib.pyplot as plt
          import matplotlib as mpl
          import numpy as np
          from astropy import constants as const
          from astropy import units as u
          import matplotlib.ticker as ticker

          t=np.linspace(0,183000,6400)
          M_lens=0.5 # solar masses
          DL=1.0 # kpc
          DS=8.0 # kpc
          vel=100 # km/s
          y0=np.linspace(1.0,0.1,10)*2.0*np.sqrt(2.0)
          t0=np.amax(t)/2.0 # days

          def tterm(M,DL,DS,v,t0,t):
              tE=EinsteinCrossTime(M,DL,DS,v).value
              return ((t-t0)/tE)

          def shift_par(M,DL,DS,v,y0,t0,t):
              tt=tterm(M,DL,DS,v,t0,t)
              yy=np.sqrt(y0**2+tt**2)
              return(tt/(yy**2+2))

          def shift_per(M,DL,DS,v,y0,t0,t):
              tt=tterm(M,DL,DS,v,t0,t)
              yy=np.sqrt(y0**2+tt**2)
              return(y0/(yy**2+2))

          def EinsteinCrossTime(M,DL,DS,v):
              theta_e=theta_e_func(M,DL,DS)
              return(((theta_e.to(’radian’).value*DL*u.kpc).to(’km’)/v/u.km*u.s).to(’day’))

          def theta_e_func(M,DL,DS):
              mass=M*const.M_sun#.value
              G=const.G#.value
              c=c=const.c#.value
              aconv=180.0*3600.0/np.pi*u.arcsecond
              return((np.sqrt(4.0*(G*mass/c/c).to(’kpc’)*(DS-DL)/DL/DS/u.kpc))*aconv)

          tt=tterm(M_lens,DL,DS,vel,t0,t)
          thetae=theta_e_func(M_lens,DL,DS)*1.e6

          fig,ax=plt.subplots(1,1,figsize=(10,10))

          color=iter(cm.rainbow(np.linspace(0,1,y0.size)))

          for i in range(y0.size):
              c=next(color)
              sx=shift_par(M_lens,DL,DS,vel,y0[i],t0,t)
              sy=shift_per(M_lens,DL,DS,vel,y0[i],t0,t)
              ax.plot(tt,np.sqrt(sx**2+sy**2)*thetae,’-’,c=c)
          ax.set_xlim([-1000,1000])
          ax.set_ylim([0.0,220])
          ax.xaxis.set_tick_params(labelsize=20)
          ax.yaxis.set_tick_params(labelsize=20)

          ax.set_xlabel(’$(t-t_0)/t_E$’,fontsize=20)
          ax.set_ylabel(’$\delta$ [$\mu\mathrm{as}$]’,fontsize=20)

          xx=[-1000,1000]
          yy=[2,2]
          ax.plot(xx,yy,’--’)
          ax.text(-990,7,’$\sim 100\mathrm{yr}$’,horizontalalignment=’left’,fontsize=20)

          xx=[-1000,1000]
          yy=[20,20]
          ax.plot(xx,yy,’--’)
          ax.text(-90,25,’$\sim 10\mathrm{yr}$’,horizontalalignment=’right’,fontsize=20)

          xx=[-1000,1000]
          yy=[200,200]
          ax.plot(xx,yy,’--’)
          ax.text(-10,205,’$\sim 1\mathrm{yr}$’,horizontalalignment=’right’,fontsize=20)
```

```
Out[101]: <matplotlib.text.Text at 0x1253c3710>
```


$\sim 1\mathrm{yr}$
$\sim 10\mathrm{yr}$
$\sim 100\mathrm{yr}$
δ [μas]
200
150
100
50
0
−1000
−500
0
500
1000
$(t - t_0)/t_E$